BEGINNING ALGEBRA

The Nursing Department of Labouré College requested the Department of Academic Planning and Support Services to help with mathematics preparatory materials for its Bachelor of Science (BSN) students who would soon start a course in statistics. That was in the fall semester of 2012. The Department of Academic Planning and Support Services was not ready, as they were ready with basic mathematics for students pursuing the Associate of Science degree. That request gave birth to what we initially called "The Algebra Project". The algebra project eventually evolved to "Beginning Algebra".

Beginning Algebra is a review of the rudiments of algebra necessary to pursue a basic course in statistics. It deals with evaluation of algebraic expressions; addition and subtraction of algebraic expressions; multiplication of algebraic expressions; division of algebraic expressions; algebraic equations (linear equations, inequalities and quadratic equations); factoring (prime factors, common monomial factors, common binomial factors, factoring of simple trinomials, and factoring of more complicated trinomials).

The pedagogy is simple. The review is organized in sections and subsections. Each section or subsection begins with definitions of basic principles and progresses to a brief discussion of the principles by use of sample problems. At the end of each section, a careful selection of practice exercises is provided to test the comprehension of the student. Finally, the answers to the practice exercises are found at the end of the review as an appendix.

ACKNOWLEDMENTS

In the course of writing this review, I have used the following books: (1) Pre-Algebra by K. Elayn Martin-Gay, 2nd Edition. (2) Intermediate Algebra For College Students by Thurman S. Peterson and Charles R. Hobby, 6th Edition. (3) College Algebra by Robert Blitzer, Instructor's Edition. These publications provided the general background to the discussions. Some examples exercises were directly extracted from these books, but the methodology of presentation, which tends to be conversational, is mine. I personally worked through all the sample problems and practice exercises and made sure the answers provided at the end of the review are correct.

SECTION I - EVALUATION OF ALGEBRAIC EXPRESSIONS

Definitions:
Algebraic Expressions: An algebraic expression is a combination of numbers and letters which are connected by any number of the standard mathematical operations – addition, subtraction, multiplication, and division. An algebraic expression is made up of parts. The parts that are separated by addition and subtraction signs are called terms.

Monomial: A monomial is an algebraic expression that is made up of one term

Binomial: It is an algebraic expression composed of two terms.

Trinomial: A trinomials is an algebraic expression that is composed of three terms.

Polynomial: It is an algebraic expression made up of many terms.

Examples: 2a is a monomial
2a + b is binomial
2a + b – c is trinomial
2a + b – c – d is called quadrinomial.

Product: When two or more numbers or letters are multiplied, the result is called a product. For example, a . b = ab;
2 . c = 2c
3 . b . c = 3bc
Each of the results, ab; 2c; and 3bc is a product.

Factor: The quantities used in multiplication to obtain products are called factors. For example: in a . b = ab, a and b are factors; in 2 . c = 2c, 2 and c are factors; while in 3 . b . c = 3bc, 3, b, and c are factors.

Numerical Coefficient: When a factor contains a number, that number is called the numerical coefficient of the factor. For example, 3 is the numerical coefficient of 3bc. In practice, when there is no numerical coefficient, we assume the coefficient is 1. Thus, in solving problems, we assume that the numerical coefficient of ab is 1. Therefore, in practice, every factor has a numerical coefficient.

Exponents: Let us consider the following algebraic expression:

$2x^5 + x^4 + x^3 + x^2 + x.$

The numbers 5, 4, 3, and 2 are exponents. There can be negative exponents as well.

Polynomial: A polynomial is an algebraic expression that is made up of many terms. Specifically, it is made up of all the exponents of its terms are positive. Thus, the algebraic expression, $2x^5 + x^4 + x^3 + x^2 + x$., is a polynomial.

The Degree of a polynomial: The degree of a polynomial is the highest exponent of any of its terms. Thus the degree of $2x^5 + x^4 + x^3 + x^2 + x$. is 5, since 5 is the highest exponent of any of its terms.

Sample problems:

Problem I - 1. Explain the difference between 3b and b^3

Solution I - 1. 3b is the same as

$3.b = 3b$

and

$b^3 = b.b.b$

If we arbitrarily assign a value of 2 to b, we can obtain the following results:

$3.2 = 6$

and

$2.2.2 = 8$

Problem I - 2. If x = 1, what is the value of x^4 ?

Solution I - 2. Substitute x for 1. In other words, replace x with 1. Thus x^4 becomes

$1.1.1.1 = 1.$

Problem I - 3. If x = 2, what is the value of x^4 ?

Solution I - 3. Replace x with 2. Hence, x^4 becomes $2.2.2.2 = 16$.

Problem I - 4. Evaluate $by + cx + dz – dv$, if $b = 1$; $y = 2$; $c = 3$; $x = 4$; $d = 5$; $z = 6$, and $v = 7$.

Solution I - 4. Replace the letters with the numerical values as follows:

by becomes $1 . 2 = 2$
cx becomes $3 . 4 = 12$
dz becomes $5 . 6 = 30$
dv becomes $5 . 7 = 35$

Therefore $by + cx + dz – dv = 2 + 12 + 30 – 35 = 44 – 35 = 9$

Very often we are required to evaluate algebraic expressions, given numerical values as in problem 4 above. Some times the expressions we are required to evaluate are complicated. It is thus important to consider more examples.

Problem I - 5. If $x = 3$; $y = 4$ and $z = 2$, evaluate $5(x + y)(y – z)$

Solution I - 5. We begin by replacing the letters with the assigned numerals.
$5(x + y)(y – z)$ becomes $5(3 + 4)(4 – 2)$
$= 5 . 7 . 2 = 70$

Problem I - 6. Evaluate $3(ab – cd) + 4(ab + cd)$, given the following numerical values:
$a = 5$; $b = 4$; $c = 3$; and $d = 2$

Solution I - 6. As usual, we replace all letters with numerical values
$3(ab – cd) + 4(ab + cd)$ becomes $3(5 .4 – 3 . 2) + 4(5 . 4 + 3 . 2)$
This becomes $3(20 – 6) + 4(20 + 6)$
$= 3 . 14 + 4 . 26$
$= 42 + 104$
$= 146$

Problem I - 7. Evaluate $3(ab – cd) – 4(ab + cd)$, if the values of a, b, c, and d are 5, 4, 3, and 2, respectively.

Solution I - 7. It is very important to pay attention to the negative sign separating $3(ab –cd)$ from $4(ab + cd)$. Let us now do our substitution. We now have;
$3(5 . 4 – 3 . 2) – 4(5 . 4 + 3 . 2)$
$= 3(20 – 6) – 4(20 + 6)$
$= 3 . 14 – 4 . 26$
$= 42 – 104 = -62$.

Practice Exercises I

1. Simplify the following expressions:

   (a) $3(4x – 6)$ (b) $5y – 4(3y – 2)$ (c) $3(2x+ 3y) + 6(x + y)$

2. Express each of the following in exponential form:

   (a) 5xyy. (b) zzzzzz (c) 6aaaddd (d) xxyyhhh (e) 2yy

3. Evaluate the following when a is 2 and b is 3:

   (a) b^4 (b) $2ab$ (c) $5a – 2b$ (d) $-2b + 5a$ (e) $4b^4$

4. Evaluate $3x^4 - 4x^3 + 2x^2 + 7x – 3$ when $x = -1$

5. Evaluate $\frac{1}{x} + \frac{1}{y}$ when x = -3 and y = 3

6. Evaluate x - $\frac{3}{x}$ + 1 if x = 4

7. Evaluate x^2 - y if x = 3 and y = 4

8. Find $x^3 - y^3$ when x = 3 and y = 2

SECTION II - ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

It is desirable at this point to make a few more definitions.

Similar or like terms: These are defined as terms that differ only in their numerical coefficients. Let us look at the following polynomials:

(a) 3x + 2x - x.. We can see that all the terms are x.

(b) 3xy + 2xy - xy. We can equally see that all the terms are xy.

(c) 3xy + 2xz - 2xy + xz. In this case, there are two different terms, xy and xz.

(d) 3xy - 2xz + 2xy + xz

In case (a), since all the terms are like terms, we simply do our addition and subtraction by adding and subtracting the numerical coefficients of the terms. Thus 3x + 2x – x becomes 4x. (Notice that we do 3 +2 – 1 = 4. Then we affix the term x at the end).
In case (b), we follow the same logic: 3xy + 2xy – xy, the similar term is xy. We therefore just add and subtract the numerical coefficients 3, 2, and -1. The result is 4xy.
Case (c) presents a slightly different scenario because there are two terms that are not like terms, xy and xz. To solve (c), we must first of all group like terms together by introducing parentheses or brackets. This process is also known as **collecting like terms** together. Thus, 3xy + 2xz – 2xy + xz becomes (3xy – 2xy) + (2xz + xz) = xy + 3xz
In case (d), 3xy – 2xz + 2xy + xz becomes (3xy + 2xy) – (2xz + xz) = 5xy – xz. The following working problems will illustrate most intricacies involved in inserting and removing parentheses or brackets.

Sample Problems:

Problem II – 1. Remove parentheses and collect terms in the following expression:

$$(7x – 4) + (x – 5)$$

Solution II – 1. First of all, re-write (7x – 4) + (x – 5) as

$1.(7x – 4) + 1.(x -5)$
$= 7x – 4 + x – 5$
Collect terms: $= 7x + x -4 -5$
$= 8x – 9$

Problem II – 2. Remove parentheses are collect terms in the following expression:

$$(5x – 3y) –(2x – 4y)$$

Solution II – 2. As usual, we start by re-writing (5x – 3y) – (2x – 4y) as

$1.(5x – 3y) – 1. (2x – 4y)$
$= 5x – 3y – 2x + 4y$
Collect terms: $= 5x – 2x – 3y + 4y$
$= 3x + y$

Problem II – 3. Remove parentheses and collect terms in the following expression:

$$(a^2 + 2b^2) – (3a^2 + 2ab - b^2) + (3ab – 5b^2)$$

Solution II – 3. Begin by re-writing $(a^2 + 2b^2) – (3a^2 + 2ab - b^2) + (3ab – 5b^2)$ as

$$1.(a^2 + 2b^2) – 1.(3a^2 + 2ab - b^2) + 1.(3ab – 5b^2)$$

Remove parentheses: $= a^2 + 2b^2 – 3a^2 - 2ab + b^2 + 3ab – 5b^2$
Collect terms: $= a^2 – 3a^2 + 2b^2 + b^2 - 5b^2 - 2ab + 3ab$
$= -2a^2 - 2b^2 + ab$

We learn two facts in solution II – 3. (1). When no numerical value is assigned in front of parentheses or brackets, we assume that value is 1. (2). When a negative sign precedes parentheses or brackets, the sign inside the parentheses changes (a negative becomes a positive and a positive becomes a negative) upon opening the parentheses.

Problem II – 4. Remove parentheses and collect terms of the following expression:

$$x – [3x – (2x – y)]$$

Solution II – 4. In this case, there is a set of parentheses and a set of brackets. We start by removing the set that is inner- most of the expression. In our present example, we should remove the parentheses before the brackets. The expression x – [3x –(2x – y)] becomes

$$x – [3x – 2x + y]$$

Then remove the brackets: $x – [3x – 2x + y]$ becomes

$$x – 3x + 2x – y$$

Collect terms: $x – 3x + 2x – y = 3x – 3x – y = -y$

Practice Exercises II

1. $3a – 7a$
2. $3b^2 + 2^2 - 4^2$
3. $2z – 3z + z – 4z$
4. $-5b^2 + 2b^2$
5. $- x^3 +2 x^3 + 3 x^3$
6. $by – 4by – 2by + 3by$
7. $2x^2 - xy – 3y^2 + x^2 + xy - y^2$
8. $2a – 3b – c – 4a + 2b + 3c – a + b + c$

Remove parentheses and combine like terms:

9. $(2a – b) – (a + 4b)$
10. $(2x –y) + (- x + y) + (- 3x + 2y)$
11. $(6u + 3v – z) – (2u – v + 2z) + (u – 3v + z)$
12. $6x + y[(3x – 5y) – (4x + 2y)]$
13. Subtract the sum of $2x^3 + 3x – 1$ and $2x – 7x^2$ from the polynomial $3x^2 - x + 7$
14. Find the sum of $2x – y + 3z$ and $2y – 4z$ and subtract the result from $– z + 2y$
15. What is the negative of the polynomial $– 2x^3 + 3x^2 - 5x + 4$? In other words, what polynomial must be added to it to make it zero?

16. Subtract $- x^2 - y^2$ from zero and add the difference to the sum of $3y^2 - x^2$ and $xy + x^2 - y^2$

SECTION III – MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Let us start by learning the law of exponents for multiplication. By definition,

b^4 = bbbb and b^7 = bbbbbbb and $b^4 . b^7 = b^{4+7}$

In general, $b^x . b^y = b^{x+y}$

This is called the law of exponents for multiplication.

Sample Problems:

Problem III – 1. Multiply $2c^3$ by $3c^5$

Solution III – 1. Apply the law of exponents for multiplication:

$$2c^3 . 3c^5 = (2 . 3)c^{3+5} = 6c^8$$

Problem III – 2. Multiply $4x^2y^3z^4$ by $5x^3y^4z^5$

Solution III – 2. Apply the law of exponents for multiplication:

$$\begin{aligned} 4x^2y^3z^4 \ . \ 5x^3y^4z^5 &= (4 . 5)x^{2+3}y^{3+4}z^{4+5} \\ &= 20x^5y^7z^9 \end{aligned}$$

Note: You should note that in solution III – 2, the exponents of one letter cannot be used in any way with those of another letter.

Problem III – 3. What is the product of $-2x^2y^3$, $3y^2z$, and $-xyz$?

Solution III – 3 . Let us apply the law of exponents for multiplication:

The product of $-2x^2y^3$, $3y^2z$, and $-xyz$ is

$$\begin{aligned} &(-2x^2y^3)(3y^2z^1)(-x^1y^1z^1) \\ &= (-2)(3)(-1)\ x^{2+1}y^{3+2+1}z^{1+1} \\ &= 6x^3y^6z^2 \end{aligned}$$

Problem III – 4. What is the product of $2xy^2 + 3x^2y - 4$ and $-2xy$?

Solution III – 4. In this case we are multiplying a polynomial by a monomial, but the law of law of exponents for multiplication is obeyed. We re-write the problem:

$$(-\ 2xy)(2xy^2 + 3x^2 y - 4)$$

We next open the parentheses by multiplying (-2xy) by each term of the polynomial ($2xy^2 + 3x^2 y - 4$). The sum of each product is:

$$\begin{aligned}
&(-\ 2xy)(\ 2xy^2\ ) + (-\ 2xy)(\ 3x^2\ y) - (4)(-2)(\ xy)\\
&= (-\ 4)(x^{1+1} y^{1+2}) + (-6)(x^{1+2} y^{1+1}) + (8)(xy)\\
&= -\ 4x^2 y^3 - 6x^3 y^2 + 8xy
\end{aligned}$$

Problem III – 5. Multiply 2x – 3 by 3x + 2

Solution III – 5. We multiply each of the first two terms by each of the second to terms:

$$\begin{aligned}
(2x - 3)(3x + 2) &= (2x)(3x) + (2x)(2) + (-\ 3)(3x) + (-\ 3)(2)\\
&= 6x^2 \qquad + 4x - 9x - 6\\
&= 6x^2 - 5x - 6
\end{aligned}$$

Problem III – 6. Multiply $3x^2 - 5x - 2$ by 3x – 4

Solution III – 6. In this example, we shall arrange our work so that the method of multiplying polynomials is similar to the method of multiplying ordinary integers. We do it like this:

$$\begin{array}{l}
3x^2 - 5x - 2\\
\qquad 3x - 4\\
\hline
9x^3 - 15x^2 - 6x\\
\qquad -\ 12x^2 + 20x + 8\\
\hline
9x^3 - 27x^2 + 14x + 8
\end{array}$$

It should be noted that after multiplying by the individual terms, we place all the terms involving the same power of x in the same column. The product is obtained by adding each column and forming the sum of the resulting terms.

Practice Exercises III

Solve the following:

(1) $(2x)(-3x)$

(2) $(-4x)(-x^2)$

(3) $(2y)(-z)(-5yz)$

(4) $2a^2b(3a - 9b^2 - ab)$

(5) $-2y(3x - 2y + xy - 5)$

(6) $(2a + 5)(a - 4)$

(7) Multiply $(2x^2 - x + 3)$ by $7x - 5$

(8) Multiply $(-x + 5)$ by $(5x^2 - 3x + 1)$

(9) Multiply $(x^2 - y^2)(-x^4 + 3x^2y^2 - 4b^4)$

SECTION IV – DIVISION OF ALGEBRAIC EXPRESSIONS

Let us consider a division property called the cancellation property. It states that in a division operation, we may cancel factors that are common to both the dividend and the divisor. Take $\frac{x^6}{x^3}$ as an example. This can be re-written as $\frac{xxxxxx}{xxx}$. We see that can see that x appears 6 times in the dividend and 3 times in the divisor. We can therefore cancel 3 xs in the divisor against 3 xs in the dividend. We are thus left with 3 xs in the dividend and our result is $xxx = x^3$. Similarly, $\frac{x^3}{x^6} = \frac{xxx}{xxxxxx} = \frac{1}{xxx} = \frac{1}{x^3}$

We can then make a general statement that if n and m are integers, $\frac{x^n}{x^m} = x^{n-m}$ where $x \neq 0$. In the special case when n = m, $\frac{x^n}{x^m} = x^{m-m} = x^0$, when $x \neq 0$. Since $\frac{x^m}{x^m} = 1$, it follows that $x^0 = 1$. We thus come up with the rule that any none-zero quantity raised to the power zero is equal to one.

Let us now divide $35a^3b^3$ by $7ab^4$. We can re-write the problem as $\frac{35a^3b^3}{7ab^4}$.

To solve the problem, we have to factorize the dividend and divisor like this:

$$\frac{5.7.aaa.bbb}{7.a.bbbb} = \frac{5.aa}{1.b} = \frac{5a^2}{b} = 5a^2 b^{-1}$$

It is evident that in each case, the exponent of any letter in a quotient is the difference obtained by subtracting its exponent in the denominator from its exponent in the numerator. This is called the law of exponents for division.

Sample Problems

Problem IV – 1. Divide $42x^4 y^3 z^6$ by $-7x^2 yz^3$

Solution IV – 1. Start by re-writing the problem: $\frac{42x^4 y^3 z^6}{-7x^2 yz^3}$

Then factorize as follows: $\frac{6.7.xxxx.yyy.zzzzzz}{-7.xx.y.zzz} = \frac{6.xx.yy.zzz}{-1} = -6x^2 y^2 z^3$

Problem IV – 2. Divide $-28a^3 b^2$ by $-4ab^2$

Solution IV – 2. Re-write the problem: $\frac{-28a^3 b^2}{-4ab^2}$

Then factorize: $\frac{-4.7.aaa.bb}{-4.a.bb} = 7.aa = 7a^2$

Problem IV – 3. Divide $6x - 9y$ by -3

Solution IV – 3. On re-writing, the problem becomes $\frac{6x}{-3} + \frac{-9y}{-3} = -2x + 3y$

Practice Exercises IV.

(1) $12x \div (-4)$

(2) $-10b^2 \div 5b$

(3) $16x^2 y \div (-4xy)$

(4) $\frac{48x^5}{-6x^5}$

(5) $\dfrac{81x^3y^2z}{27x^2z}$

(6) $(2x^3 - x^2) \div (-x^2)$

(7) $\dfrac{20x^4 - 8x^2}{-4x^2}$

(8) $\dfrac{-60a^3b^2 + 12a^4b}{-12a^3b}$

(9) $(10mn^3 - 5m^2n^2 - 15m^3n^2) \div 5mn^2$

(10) $(a^2bc - ab^2c + abc) \div (-abc)$

(11) Divide $6x^2 + 7x + 2$ by $3x + 2$

(12) Divide $12m^2 - mn - 6n^2$ by $4m - 3n$

(13) Divide $6a^4 - a^2 - 12$ by $2a^2 - 3$

SECTION V. EQUATIONS

Let us consider these two relationships: (a) 5 + 4 = 9 and (b) 7 + 2 = 6. Relationship (a) can be labeled true, while relationship (b) can be labeled false. This is so because they are statements of fact. Let us now consider other relationships (c) x + 4 = 9 and (d) 7 + y = 6. Neither relationship can be labeled true or false because neither is a statement of fact. However, either relationship may be true or false depending on specific values assigned to x or y. In algebra, x and y are called variables or unknowns. Any letter of the alphabet can be used as variables. Letters of the Greek alphabet are also used extensively in algebra. The relationships (c) and (d) are called equations in which x and y are variables or unknowns

There are many types of equations. They include linear, simultaneous, quadratic, and differential equations. In order to understand what an equation is, it is important to imagine working with a scale balance. The scale balance must always be balanced. If you add a weight to the left hand side, you must add an equal amount of weight to the left to return to a balanced position. Similarly, if you subtract a weight from the right hand side, you must return to balance by subtracting an equal amount from the left hand side. For our purposes however, we shall treat only the first three.

SECTION V(a) – 1. LINEAR EQUATIONS

Linear equations are also called First – Degree Equations. The best way to learn to solve equations is to start with concrete examples. We should then start with the following examples:

Sample Problems

Problem V(a) – 1. Solve for x: x + 8 = 10

Solution V(a) – 1. Write down the equation:

$$x + 8 = 10$$

We can solve this problem by separating the variable from the constants. We traditionally keep the variables on the right hand side of the sign of equality and the constants on the right hand side. In our present situation, we do so by subtracting 8 from both sides:

$$x + 8 – 8 = 10 – 8$$

We obtain x + 0 = 2

Therefore x = 2.

Problem V(a) – 2. Solve for x: - 3x – 8 = 4

Solution V(a) – 2:

-3x – 8 = 4
-3x - 8 + 8 = 4 + 8
-3x – 0 = 12

Divide both sides by -3:

$$\frac{-3x}{-3} = \frac{12}{-3} =$$

Therefore x = -4.

Alternatively, the problem can be solved like this:

$$-3x – 8 = 4$$

Gather all the unknowns (variables) to the left hand side of the equation and all the knowns (constants) to the right. In doing so, make sure that the negative sign (-) changes to positive, while the positive sign (+) changes to negative, upon crossing the sign of equality. In our present problem, -3x – 8 becomes -3x = 4 + 8 because -8, on crossing the sign of equality, become +8 or simply 8. Divide both sides by -3:

$$\frac{-3x}{-3} = \frac{12}{-3}$$

Therefore x = -4.

When solving algebraic equations in general, we follow the following principles and proceed through the following steps:

(a) Gather all the unknowns (variables) to the left hand side of the equation; and all the knowns (constants) to the right.
(b) When a variable or a constant crosses the sign of equality, its sign changes. If the sign was negative (-), it becomes positive (+). If it was positive (+), it becomes negative (-).
(c) If you find it necessary to divide the left hand side of the equation by an entity, you must also divide the the left hand side by the same entity. Similarly, if you deem it fit to multiply the left hand side by a given entity, you must also multiply the right by that entity.

From now on, we shall solve our problems by this second alternative.

Problem V(a) – 3. Solve for x: 3x – 5 = 4x + 2

Solution V(a) – 3. Gather all the unknowns (variables) to the left hand side of the equation; and all the knowns (constants) to the right. Make sure the negative signs become positive, and the positive signs become negative upon crossing the sign of equality:

Thus 3x – 5 = 4x + 2 becomes:

3x – 4x = 2 + 5

And

-x = 7.

To obtain x, we must divide both sides by -1:

$$\frac{-x}{-1} = \frac{7}{-1}$$

Therefore x = -7.

Problem V (a) – 4. Solve for x: 5(2x – 3) + 7 = - 6(4 – 2x) – 5

Solution V (a) – 4. Open the parenthesis or expand the enclosed terms of the equations:

10x – 15 + 7 = -24 + 12x – 5

Gather all variables to the left and all constants to the right:

10x – 12x = - 24 – 5 + 15 – 7 [notice the change in signs]

Then, - 2x = - 21

Divide both sides by – 2:

$$\frac{-2x}{-2} = \frac{-21}{-2}$$

Therefore x = $\frac{21}{2}$ and x = 10.5

Problem V a) – 5. Solve for x: $(2x – 5)^2 - (x + 1)^2 = 3x^2 + 4x – 2$

Solution V(a) – 5. Open the parenthesis of each term:
$(2x – 5)^2 = (2x – 5)(2x - 5) = 4x^2 - 10x – 10x + 25 = 4x^2 - 20x + 25$
$(x + 1)^2 = (x + 1)(x + 1) = x^2 + x + x + 1 = x^2 + 2x + 1$
Then, $(2x – 5)^2 - (x + 1)^2 = 4x^2 - 20x + 25 - x^2 - 2x – 1 = 3x^2 - 22x + 24$
Our equation now becomes $3x^2 - 22x + 24 = 3x^2 + 4x – 2$
Gather all variables to the left and all constants to the right:
$3x^2 - 3x^2 - 22x – 4x = - 2 – 24$
$0 – 26x = -26$ and
$-26x = -26$
Divide both sides by -26:
$$\frac{-26x}{-26} = \frac{-26}{-26}$$
Therefore $x = 1$.

Problem V(a) – 6. Solve for x: $x + 2 = x – 3$

Solution V(a) – 6. Following our rules:
$x + 2 = x – 3$
$x – x = - 3 – 2$
$0x = - 5$
$$\frac{0x}{0} = \frac{-5}{0}$$
We realize that division by 0 (zero) is not possible. Hence, the equation has no solution. We can also say the solution is an empty set.

Problem V(a) – 7. Solve for x: $x + 2 = x + 2$.

Solution V(a) – 7. Following the usual rules:
$x + 2 = x + 2$
$x – x = 2 – 2$
$0x = 0$
$$\frac{0x}{0} = \frac{0}{0}$$
$0 = 0$
In the present case, our equation is equivalent to a true equation that does not involve (x). Any such equation has the set of all real numbers as its solution set.

Practice Exercises V(a).

Solve the following problems:

1. $7x + 6 = 5x + 12$

2. $-3x - 6 + x = 4x + 4$

3. $(3x + 2)^2 - 3x(3x + 1) = 3x - 5$

4. $-2(3x - 7) = 3 - (2 - x)$

5. One number is 11 more than another, and their sum is 97. Find the numbers.

6. A stick 63 centimeters long is broken into two pieces so that one piece is six times as long as the other. How long is each piece?

7. The sum of two consecutive integers is 155. Find the integers.

8. Three times a number is 75 more than half the number. What is the number?

9. A woman is three times as old as her son, and the difference in their ages is 38 years. Find their ages.

10. The sum of four consecutive integers is 130. Find the integers.

11. To finance a fire truck costing $133,000, the state contributed twice as much as the county, and the county contributed twice as much as the city. How much did each contribute?

12. Forty-four dollars is to be divided among three girls and two boys, so that each of the girls receives three times as much as each of the boys. How much does each of the boys receive? How much does each of the girls receive?

13. In 35 years, Mary will be six times as old as she is now. How old is she now?

14. Four less than three times a certain number is the same as 28 more than twice the number. Find the number.

SECTION V (b) – INEQUALITIES

If a number b lies to the left of another number c on a number line, the number b is said to be less than number c.

___________b ____________________________c_________________

The notation $b < c$ is used to indicate that b is less than c. We can also say that $c > b$ to indicate that c is greater than b. the symbol $b \leq c$ means b is less than or equal to c. Conversely, $c \geq b$ means c is greater than or equal to b.

Inequalities of the form $4x + 4 < 2x + 8$ can be solved in the same manner we solve linear equations of the form $4x + 4 = 2x + 8$

Thus $4x - 2x = 8 - 4$

$2x = 4$

$x = 2$

Similarly, $4x + 4 < 2x + 8$

$4x - 2x < 8 - 4$

$2x < 4$

$x < 2$

In the course of solving inequality problems we shall encounter multiplication or division by negative numbers. It would be important to note that in such situations the sign of inequality changes direction. It is equally important to emphasize that multiplication or division by positive numbers changes nothing.

Example Problems:

Problem V(b) – 1. Find the solution of $-4x + 7 \leq 5x - 2$

Solution V(b) – 1. $-4x + 7 \leq 5x - 2 = -4x - 5x \leq -2 - 7 = -9x \leq -9$.

In order to solve for x, we must divide both sides of the inequality by – 9:

$$\frac{-9}{-9}x \geq \frac{-9}{-9}$$

Notice that inequality sign has changed direction. Thus, $x \geq 1$.

Problem V(b) – 2. Find the solution of $\frac{x}{2} + 5 > \frac{x}{3} + 6$

Solution V(b) – 2. $\frac{x}{2} + 5 > \frac{x}{3} + 6 = \frac{x}{2} - \frac{x}{3} > 6 - 5$

$$= \left(\frac{1}{2} - \frac{1}{3}\right)x > 1$$

$$= \frac{x}{6} > 1$$

Multiplying both sides by 6 yields x > 6.
It should be noted that multiplication by +6 does not change the direction of the inequality sign.

Practice Exercises V(b)

Solve the following:

1. $3x < 9$
2. $5x \leq -10$
3. $-2x > 4$
4. $-2x \leq -6$
5. $-2x + 3 \leq 5$
6. $-5x + 3 > -7$
7. $-x + 3 < x + 5$
8. $\frac{1}{2}x - 1 > 3$
9. $-2x + 2 \leq 3x - 8$
10. $x + 2 > -2x + 8$
11. $x - 3 \leq 2x - 5$
12. $-3x + 9 < 4 - 5x$
13. $-2x + 4 > 4 - 5x$

SECTION VI - FACTORING

SECTION VI(a) – Prime Factors

Factoring is a process in which an algebraic expression that is a product of two or more components is broken down into those components. For example, 2 and 3 are factors 6 because $2 \cdot 3 = 6$. If a natural number greater than 1 has no factors, it is called a Prime Number. Therefore the numbers 2, 3, 3, 5, 7, 11, 13, 17, 19, 23, ect, are prime numbers. A natural number that is greater than 1 but not a prime number is called a composite number. Examples are 4, 6, 8, 18, 12, etc. When a composite number is completely broken down to its prime components, we say it has been completely factored. For example, when 12 is broken down to $2 \cdot 2 \cdot 3$, we say 12 has been completely factored.

Practice Exercises VI(a)

*** The concept of prime factors will be applied in all subsequent examples. There is therefore no need to devote a whole set of practice exercises for prime factors.

SECTION VI(b) – Common Monomial Factors

We already know that in order to multiply a multinomial by a monomial, we apply the distributive axiom of multiplication. For example, $b(x + y – z) = bx + by – bz$. Conversely, if given the expression, $bx + by - bz$, we may decide to re-write it as $b(x + y – z)$. It is therefore noteworthy to observe the following general principle: If one factor occurs in every term of an algebraic expression, the expression may be factored as the product of that common factor and the quotient obtained by dividing the original expression by the factor. Thus, the first step in factoring a polynomial is to look for a monomial that is a common factor.

Example Problems VI(b)

Problem VI(b) – 1. Factor the expression: $4x – 6$

Solution VI(b) – 1. We can see clearly that of the two terms of the binomial contains the common factor, 2. We can then factor out 2 and state that:

$$4x – 6 = 2(2x – 3).$$

Problem VI(b) – 2. Factor the expression: $3ax – 3$

Solution VI(b) – 2. A cursory inspection reveals that each of the two terms contains 3. Hence, 3 is a common factor. Therefore, $3ax – 3 = 3(ax – 1)$

Problem VI(b) – 3. Factor the following expression: $x^3 - 2x^2 + 5x$

Solution VI(b) – 3. We can see that x is common in all the terms of the trinomial. Factoring out x yields:

$$x^3 - 2x^2 + 5x = x(x^2 - 2x + 5).$$

Problem VI(b) – 4. Factor this expression: $3a^4 b - 3a^2 b^2 + 6a^2 b^2$

Solution VI(b) – 4. Since 3ab is common in all the terms of the expression, we can say:

$$3a^4 b - 3a^2 b^2 + 6a^2 b^2 = 3ab(a^3 - ab + 2ab)$$

Problem VI(b) – 5. Factor the expression: $a(x + y) + b(x + y)$

Solution VI(b) – 5. Our common factor in the expression is $(x + y)$. Therefore

$$: a(x + y) + b(x + y) = (a + b)(x + y)$$

Problem VI(b) – 6. Factor: $3x^2 (4x + 1) - 5xy(4x + 1) + y(4x + 1)$

Solution VI(b) – 6. It can be seen that $(4x + 1)$ is the common factor in the expression. We factor it out and the result is:

$$3x^2 (4x + 1) - 5xy(4x + 1) + y(4x + 1) = (4x + 1)(3x^2 - 5xy + y)$$

Practice Exercises VI(b)

Factor the following:

1. $5x - 10y$
2. $16x^2 - 8x^3$
3. $3b^2 - 12b$
4. $121a^2 x^2 - 11ax$
5. $3x^3 - 2x^2 + x$
6. $4a^3 b + 6ab^2 + 2ab^3$
7. $x^3 y^3 z^3 + x^2 y^3 z^2 + x^4 y^2 z^5$

8. $2x(a - b) - 3(a- b)$

9. $3a(x + y) + 2b(x + y)$

10. $x(x + y) - y(x + y)$

11. $5m(x - y) - 2n(x - y)$

12. $2(x - y) + 3a(x - y)$

13. $- 3x(3x - y) + 2y(3x - y)$

14. $x(x + y) - y(x + y) - 3(x + y)$

SECTION VI(c) - Common Binomial Factors

When there is no factor that appears in all the terms of an expression, it is still possible to group the terms in such a way that each group will have a common factor.

Problem VI(c) - 1. Factor $ac + bc + ac + bd$

Solution VI(c) - 1. First of all, group the expression like this:

$$\begin{aligned} &(ac + bc) + (ad + bd) \\ &= c(a + b) + d(a + b) \\ &= (c + d)(a + b) \end{aligned}$$

Problem VI(c) - 2. Factor: $x^2 - ax + bx - ab$

Solution VI(c) - 2. Group terms together

$$\begin{aligned} x^2 - ax + bx - ab &= (x^2 - ax) + (bx - ab) \\ &= x(x - a) + b(x - a) \\ &= (x + b)(x - a) \end{aligned}$$

Problem VI(c) - 3. Factor: $12a^2 - 4ab - 3ac + bc$

Solution VI(c) – 3. $$\begin{aligned} 12a^2 - 4ab - 3ac + bc &= (12a^2 - 4ab) - (3ac - bc) \\ &= 4a(3a - b) - c(3a - b) \\ &= (4a - c)(3a - b) \end{aligned}$$

Problem VI(c) - 4. Factor: $c^2 + cax + c + ax$

Solution VI(c) - 4. $c^2 + cax + c + ax = (c^2 + cax) + (c + ax)$
$= c(c + ax) + 1(c + ax)$
$= (c + 1)(c + ax)$

Practice Exercises VI(c)

Factor the following expressions:

1. $a^2 - ab + ac – bc$
2. $as + bs – at – bt$
3. $4x^2 + 4xy – ax – ay$
4. $2x^4 + a^2 x^2 + 2bx^2 + a^2 b$
5. $x – y – xy + x^2$
6. $3ac – 3bc – a + b$
7. $2a^3 - a^2 b + 4ab^2 - 2b^3$
8. $3x^3 - 2x^2 + 3x – 2$
9. $3y^3 + 3y^2 - y – 1$
10. $10b^3 - 8b^2 - 15b + 12$
11. $- a^3 + 2a^2 - 5a + 10$
12. $x^3 + 1 + x^2 + x$
13. $– ax + ay – bx + by + cx – cy$
14. $6a^3 - 5 + 2a – 15a^2$

SECTION VI(d) – Factoring of Simple Trinomials

A simple trinomial is a trinomial of the form $1.x^2 + 2x – 15$ which is equal to $x^2 + 2x – 15$, in which the coefficient of x^2 is 1. Let us now consider factoring a trinomial and remember how the product of two binomials results in a trinomial. As an example, let us multiply (x + A) by (x + B). The result is $x^2 + (A + B)x + AB$. We can see that the sum of A and B, (A + B), is the coefficient of x, while the product of A and B, AB, is the constant term. Let us at this juncture factor the following: .$x^2 + 2x – 15$. If .$x^2 + 2x – 15$ is equal to (x + A)(x + B), then the sum of A and B must be 2, the coefficient of x; and the product of of A and B must be – 15, the constant term. Therefore we want A and B in such a way that A + B = 2 and AB = - 15. It is evident that such numbers are – 3 and 5. Try the following test:

- 3 + 5 = 2 (True)
- (- 3)(5) = - 15 (True)

Hence, .$x^2 + 2x – 5 = x^2 - 3x + 5x – 15$
Group the terms like this: $(x^2 - 3x) + (5x – 15)$
.Then factor each pair: $(x^2 - 3x) + (5x – 15) = x(x – 3) + 5(x – 3)$
And $x(x – 3) + 5(x – 3) = (x + 5)(x – 3)$

Sample Problems

Problem VI(d) – 1. Factor the following expression: $a^2 + 7a + 12$

Solution VI(d) – 1. We must find two numbers such that their sum is equal to 7 and their product is 12. We see that 3 and 4 fulfill the two conditions.
[Test: 3 + 4 = 7 and (3)(4) = 12]
We can now proceed: $a^2 + 7a + 12 = a^2 + 3a + 4a + 12$
Group the new terms: $(a^2 + 3a) + (4a + 12)$
$= a(a + 3) + 4(a + 3)$
$= (a + 4)(a + 3)$

Problem VI(d) – 2. Factor the following: $x^2 - x – 6$

Solution VI(d) – 2. We start by looking for two numbers such that theie sum is – 1 and their product is – 6. The numbers are – 3 and 2. Hence, $x^2 - x – 6$ can be re-written as:

$x^2 -3x + 2x – 6$
Group the terms: $x^2 - 3x + 2x – 6 = (x^2 - 3x) + (2x – 6)$
$= x(x – 3) + 2(x – 3)$
$= (x + 2)(x – 3)$

Problem VI(d) – 3. Factor: $x^2 + 5x – 104$

Solution VI(d) – 3. Let us find two numbers such that their sum is 5 and their product is - 104. The two numbers are – 8 and 13. Then we re-write $x^2 + 5x – 104$ as:
$x^2 + 13x – 8x - 104$
$= (x^2 + 13x) – (8x +104)$

$= x(x + 13) – 8(x + 13)$
$= (x – 8)(x + 13)$

Problem VI(d) – 4. Factor: $x^2 - 6xy + 8y^2$

Solution VI(d) – 4. As usual, we look for two numbers such that their sum is – 6 and their product is 8. Very easily, it can be seen that the two numbers are – 2 and – 4. We follow up by re-writing $x^2 - 6xy + 8y^2$ as:

$x^2 - 2xy – 4xy + 8y^2$
Then we group them: $(x^2 - 2xy) – (4xy - 8y^2)$
$= x(x – 2y) – 4y(x – 2y)$
$= (x – 4y)(x – 2y)$

Problem VI(d) – 5. Factor the following expression: $x^2 y^2 - 6xy + 8$

Solution VI(d) – 5. Look for two numbers that have – 6 as their sum and 8 as their product. The numbers are certainly – 2 and – 4. Then $x^2 y^2 - 6xy + 8$ becomes
$x^2 y^2 - 2xy – 4xy + 8$
$= (x^2 y^2 - 2xy) – (4xy - 8)$
$= xy(xy – 2) – 4(xy – 2)$
$= (xy – 4)(xy – 2)$

Practice Exercises VI(d)

Factor:

1. $x^2 + 3x + 2$
2. $b^2 + b – 2$
3. $c^2 - 8c + 16$
4. $x^2 - 5x – 14$
5. $m^2 + 10m + 16$
6. $y^2 - 5y – 36$
7. $a^2 - a – 20$
8. $x^2 - 7x – 18$

9. $y^2 + y - 20$

10. $x^2 - 21x + 80$

11. $a^2 + 14a - 275$

12. $x^2 - 16xy + 28y^2$

13. $x^2y^2 - 26xy - 87$

14. $x^4 - 9x^2 - 162$

15. $324 - 36m^2 + m^4$

SECTION VI(e) – Factoring of Trinomials

The method used to factor simple trinomials can also be used to factor trinomials of the general form $ax^2 + bx + c$even when a is not equal to 1. In this general form,

a = the coefficient of the quadratic term
b = the coefficient of the middle term
c = the constant term.

Sample Problems VI(e)

Problem VI(e) – 1. Factor $5x^2 + 7x - 6$

Solution VI(e) – 1. First of all, multiply the coefficient of the quadratic term by the constant term. In this case, (5)(-6) = - 30. Second, find two numbers such that their sum is 7 and their product is – 30. We find that – 3 and 10 fulfill the conditions. We can now re-write $5x^2 + 7x - 6$ as : $5x^2 + 10x - 3x - 6$
Group the terms: $5x^2 + 10x - 3x - 6$ becomes:

$$\begin{aligned} &(5x^2 + 10x) - (3x + 6) \\ &= 5x(x + 2) - 3(x + 2) \\ &= (5x - 3)(x + 2) \end{aligned}$$

Problem VI(e) – 2. Factor the following expression: $7x^2 - 19x + 10$

Solution VI(e) – 2. As usual, we find two numbers such that their sum is – 19 and their product is 70. A quick inspection reveals that the numbers are – 14 and – 5. Therefore

$$\begin{aligned} 7x^2 - 19x + 10 &= 7x^2 - 14x - 5x + 10 \\ &= (7x^2 - 14x) - (5x - 10) \\ &=7x(x - 2) - 5(x - 2) \\ &= (7x - 5)(x - 2) \end{aligned}$$

Problem VI(e) – 3. Factor: $- 5x^2 + 6x + 8$

Solution VI(e) – 3. Find two numbers which have 6 as their sum and – 40 as their product. The numbers are 10 and – 4. Hence, $- 5x^2 + 6x + 8 = - 5x^2 + 10x – 4x + 8$

$$= -1(5x^2 - 10x) – (4x – 8)$$
$$= - 5x(x – 2) -4(x – 2)$$
$$= (- 5x – 4)(x – 2)$$
$$= - 1(5x + 4)(x – 2)$$

Problem VI(e)- 4. Factor: $4x^2 - 12x + 9$

Solution VI(e) – 4. Let us look for two numbers that have as their sum – 12 and their product is 36. It is obvious that the numbers are – 6 and – 6. Therefore $4x^2 - 12x + 9$ becomes $4x^2 - 6x – 6x + 9 = (4x^2 - 6x) – (6x - 9)$

$$= 2x(2x – 3) – 3(2x – 3)$$
$$= (2x – 3)(2x – 3)$$

Problem VI(e) – 5. Factor this expression: $12m^2 + 28mn – 5n^2$

Solution VI(e) – 5. We look for two numbers such that their sum is 28 and their product is – 60. The numbers are 30 and – 2. Hence,

$$12m^2 + 28mn – 5n^2 = 12m^2 + 30mn – 2mn – 5n^2$$
$$= (12m^2 + 30mn) – (2mn + 5n^2)$$
$$= 6m(2m + 5n) – n(2m + 5n)$$
$$= (6m – n)(2m + 5n)$$

** It should be noted that trinomials of the form $ax^2 + bx + c$ cannot be factored. For example, $x^2 + x + 1$. It should also be known that if we are able to factor trinomials the way we have demonstrated, we can also solve quadratic equations. This approach is based on that fact that if a product of numbers is equal to zero, then one of the factors must be zero.

Problem VI(e) – 6. Solve: $5x^2 + 7x – 6 = 0$

Solution VI(e) – 6. As usual, we find two numbers such that their sum is 7 and their product is – 30. It is evident that 10 and – 3 fulfill the conditions. We can now replace 7x with 10x – 3x and $5x^2 + 7x – 6 = 0$ becomes $5x^2 + 10x – 3x – 6 = 0$. Hence,

$$(5x^2 + 10x) – (3x + 6) = 0$$
$$5x(x + 2) – 3(x + 2) = 0$$
$$(5x – 3)(x + 2) = 0$$

Therefore either $(5x – 3) = 0$ or $(x + 2) = 0$

If $5x – 3 = 0$

Then $5x = 3$; and $x = \frac{3}{5}$

Similarly, if $x + 2 = 0$

Then $x = - 2$.

A quadratic equation therefore has two solutions; and the solution set for the problem is $\{- 2, \frac{3}{5}\}$.

Practice Exercises VI(e)

Factor the given expressions:

1. $3x^2 + 10x + 8$
2. $3x^2 - 4x – 4$
3. $2x^2 + x – 1$
4. $4x^2 + 7x – 15$
5. $2a^2 + 3a – 5$
6. $27a^2 + 6ab – 8b^2$
7. $4m^2 - 4mn – 3n^2$
8. $6x^2 y^2 + 5xy – 4$
9. $10a^4 - 29a^2 + 10$
10. $5 + 33x – 14x^2$

Find the solution set of each of the following quadratic equations:

11. $4x^2 - 7x + 3 = 0$
12. $3x^2 + 10x + 7 = 0$
13. $2a^2 + a – 21 = 0$
14. $3a^2 + 2a – 5 = 0$

15. $2m^2 + m - 10 = 0$

16. An object is thrown upward from the top of an 80-foot building with an initial velocity of 64 feet per second. The height of the object after t seconds is given by the equation $h = -16t^2 + 64t + 80$. When will the object hit the ground?

17. A right triangle is 8ft long at its base and has a hypotenuse of 10ft. What is the height of the triangle?

Answers To Practice Exercises

Practice Exercises I

1. (a) $12x - 18$ (b) $-7y + 8$ (c) $12x + 15y$

2. (a) $5xy^2$ (b) z^6 (c) $6a^3d^3$ (d) $x^2y^2h^3$ (e) $2y^2$

3. (a) 81 (b) 12 (c) 4 (d) 4 (e) 324

4. $12x - 4x^{-1}$

5. 0

6. $4\frac{1}{4}$

7. 5

8. 19

Practice Exercises II.

1. $-4a$
2. $3b^2 - 12$
3. $-4z$
4. $-3b^2$
5. $4x^3$
6. $-2by$
7. $3x^2 - 4y^2$
8. $-3a + 3c$ or $3c - 3a$
9. $a - 5b$
10. $-2x + 2y$
11. $5u + v - 2z$

12. $6x – xy – 7y^2$

13. $– 2x^3 + 10x^2 - 6x + 8$

14. $– 2x + y$ or $y – 2x$

15. $2x^3 - 3x^2 + 5x – 4$

16. $x^2 + 3y^2 + xy$

Practice Exercises III.

1. $– 6x^2$
2. $4x^3$
3. $10y^2 z^2$
4. $6a^3 b – 18a^2 b^3 - 2a^3 b^2$
5. $– 6xy + 4y^2 - 2xy^2 + 10y$
6. $2a^2 - 3a – 20$
7. $14x^3 - 17x^2 + 26x – 15$
8. $– 5x^3 + 28x^2 - 16x + 5$
9. $- x^6 + 4x^4 y^2 - 3x^2 y^4 - 4b^4 x^2 + 4b^4 y^2$

Practice Exercises IV.

1. $– 3x$
2. $– 2b$
3. $– 4x$
4. $– 8$
5. $3xy^2$

6. $1 – 2x$

7. $2 – 5x^2$

8. $5b – a$

9. $2n – m – 3m^2$

10. $– (a – b + 1)$ or $(- a + b – 1)$

11. $2x + 1$

12. $3m + 2n$

13. $13a^2 + 4$

Practice Exercises V(a)

1. 3

2. $-\frac{5}{3}$ or $1\frac{2}{3}$

3. $-\frac{3}{2}$ or $1\frac{1}{2}$

4. $\frac{13}{7}$ or $1\frac{6}{7}$

5. 43 and 54

6. 9cm and 54cm

7. 77 and 78

8. 30

9. 19 years and 57 years

10. 31 ; 32 ; 33 ; and 34

11. (State = \$76,000; County = \$38,000; City = \$19,000)

12. (Each girl = \$12.00; each boy = \$4.00)

13. 7 years

14. 32

Practice Exercises V(b)

1. $x < 3$
2. $x \leq -2$
3. $x < 2$
4. $x \leq 3$
5. $x \geq 1$
6. $x > 2$
7. $x > 1$
8. $x > 8$
9. $x \leq 2$
10. $x > 2$
11. $x \leq 1$
12. $x < \frac{5}{2}$
13. $x > 0$

Practice Exercises VI(a)

*** No need for practice exercises

Practice Exercises VI(b)

1. $5(x - 2y)$

2. $8x^2(2 - x)$

3. $3b(b - 4)$

4. $11ax(11ax - 1)$

5. $x(3x^2 - 2x + 1)$

6. $2ab(2a^2 + 3b + b^2)$

7. $x^2y^2z^2(xyz + y + x^2z^3)$

8. $2x - 3)(a - b)$

9. $(3a + 2b)(x + y)$

10. $(x - y)(x + y)$

11. $(5m - 2n)(x - y)$

12. $(2 + 3a)(x - y)$

13. $(- 3x + 2y)(3x - y)$ or $(2y - 3x)(3x - y)$

14. $(x - y - 3)(x + y)$

Practice Exercises VI(c)

1. $(a + c)(a - b)$

2. $(s - t)(a + b)$

3. $(4x - a)(x + y)$

4. $(x^2 + b)(2x^2 + a^2)$

5. $(x + 1)(x - y)$

6. $(3c - 1)(a - b)$

7. $(a^2 + 2b^2)(2a - b)$

8. $(x^2 + 1)(3x - 2)$

9. $(3y^2 - 1)(y + 1)$

10. $(2b^2 - 3)(5b - 4)$

11. $(a^2 + 5)(2 - a)$

12. $(x^2 + 1)(x + 1)$

13. $(y - x)(a + b - c)$

14. $(2a - 5)(3a^2 + 1)$

Practice Exercises VI(d)

1. $(x + 1)(x + 2)$
2. $(b - 1)(b + 2)$
3. $(c - 4)(c - 4)$
4. $(x + 2)(x - 7)$
5. $(m + 2)(m + 8)$
6. $(y + 4)(y - 9)$
7. $(a + 4)(a - 5)$
8. $(x + 2)(x - 9)$
9. $(y + 4)(y - 5)$
10. $(x - 5)(x - 16)$
11. $(a + 25)(a - 11)$
12. $(x - 14y)(x - 2y)$
13. $(xy - 29)(xy + 3)$
14. $(x^2 - 18)(x^2 + 9)$
15. $(18 - m^2)(18 - m^2) = (18 - m^2)^2$

Practice Exercises VI(e)

1. $(3x + 4)(x + 2)$
2. $(3x + 2)(x – 2)$
3. $(2x – 1)(x + 1)$
4. $(4x – 5)(x + 3)$
5. $(2a + 5)(a – 1)$
6. $(3a + 2b)(9a – 4b)$
7. $(2m + n)(2m – 3n)$
8. $(2xy – 1)(3xy + 4)$
9. $(2a^2 - 5)(5a^2 - 2)$
10. $(5 – 2x)(1 + 7x)$
11. $\{\frac{3}{4} ; 1\}$
12. $\{1 ; \frac{7}{3}\}$
13. $\{3 ; \frac{7}{2}\}$
14. $\{1 ; \frac{-5}{3}\}$
15. $\{2 ; \frac{-5}{2}\}$